K23P 3115

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBCSS – OBE – Regular) Examination, October 2023
(2023 Admission)
MATHEMATICS
MSMAT01C05 : Ordinary Differential Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks.

1. a) Determine the nature of the point x = 0 for the equation $x^3y'' + (\sin x)y = 0$.

   b) Show that $e^x = \lim_{b\to\infty} F\left(a, b, a, \frac{x}{b}\right)$.

2. Verify that the confluent hypergeometric equation $xy'' + (c - x)y' - ay = 0$ has $x = \infty$ as an irregular singular point.

3. State the Rodrigues' formula and find $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$.

4. State Bessel Expansion theorem and compute the Bessel series of the function f(x) = 1 for the interval $0 \le x \le 1$.

5. Define the Wronskian, W(t), of two solutions on [a, b] of a homogeneous linear system $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ and show that W(t) is either identically zero or nowhere zero on [a, b].

6. Solve the IVP $y' = x + y$, $y(0) = 1$ by the method of successive approximations and compare with the exact solution. **(5×4=20)**

P.T.O.

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks.

7. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x. Can you express this solution by means of elementary functions ?

8. By extending the Gamma function, give a reasonable and useful meaning to p!, when the non-negative real number p is not an integer.

9. State and prove Sturm separation theorem and then show that the zeros of the functions asinx + bcosx and csinx + dcosx are distinct and occur alternately whenever $ad - bc \neq 0$.

10. Let u(x) be any non-trivial solution of $u'' + q(x)u = 0$, where $q(x) > 0$ for all $x > 0$. If $\int_1^{\infty} q(x)dx = \infty$, then show that u(x) has infinitely many zeroes on the positive x-axis.

11. Find the general solution of the system $\begin{cases} \dfrac{dx}{dt} = 5x + 4y \\ \dfrac{dy}{dt} = -x + y \end{cases}$. **(3×7=21)**

PART – C

Answer **any three** questions from this Part. **Each** question carries **13** marks.

12. Find two independent Frobenius series solutions of $xy'' + 2y' + xy = 0$.

13. Find the general solution of the Gauss Hypergeometric equation $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$ where a, b and c are constants.
    a) near the singular point $x = 0$, when c is not an integer.
    b) near the singular point $x = 1$, when $c - a - b$ is not an integer.

14. State and prove the orthogonality property of Legendre polynomials.

15. a) Obtain $J_p(x)$, the Bessel function of the first kind of order p.
    b) Show that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$ and $J_{-\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x$.

16. State and prove Picard's Theorem. **(3×13=39)**
